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# 5–9 Factoring Pattern for $ax^2 + bx + c$

**Objective:** To factor general quadratic trinomials with integral coefficients.

## Patterns

**Factoring pattern for $ax^2 + bx + c$:** $(px + r)(qx + s)$.

**Example 1** Factor $2x^2 - 3x - 9$.

**Solution**

*Clue 1* Because the trinomial has a negative constant term, one of $r$ and $s$ will be negative and the other will be positive.

*Clue 2* You can list the possible factors of the quadratic term, $2x^2$, and the possible factors of the constant term, $-9$.

| Factors of $2x^2$ | Factors of $-9$ | |
|---|---|---|
| $2x, x$ | $1, -9$ | $-1, 9$ |
| | $3, -3$ | $-3, 3$ |
| | $9, -1$ | $-9, 1$ |

Make a chart to test the possibilities to see which produces the correct linear term, $-3x$.

Since $(2x + 3)(x - 3)$ gives the correct linear term, $2x^2 - 3x - 9 = (2x + 3)(x - 3)$.

| Possible factors | Linear Term |
|---|---|
| $(2x + 1)(x - 9)$ | $(-18 + 1)x = -17x$ |
| $(2x + 3)(x - 3)$ | $(-6 + 3)x = -3x \leftarrow$ |
| $(2x + 9)(x - 1)$ | $(-2 + 9)x = 7x$ |
| $(2x - 1)(x + 9)$ | $(18 - 1)x = 17x$ |
| $(2x - 3)(x + 3)$ | $(6 - 3)x = 3x$ |
| $(2x - 9)(x + 1)$ | $(2 - 9)x = -7x$ |

**Example 2** Factor $10x^2 - 11x + 3$.

**Solution**

*Clue 1* Because the trinomial has a positive constant term and a negative linear term, both $r$ and $s$ will be negative.

*Clue 2* List the factors of the quadratic term, $10x^2$, and the negative factors of the constant term, 3.

| Factors of $10x^2$ | Factors of 3 |
|---|---|
| $x, 10x$ | $-3, -1$ |
| $2x, 5x$ | $-1, -3$ |

Test the possibilities to see which produces $-11x$. Since $(2x - 1)(5x - 3)$ gives the correct linear term, $10x^2 - 11x + 3 = (2x - 1)(5x - 3)$.

| Possible factors | Linear term |
|---|---|
| $(x - 3)(10x - 1)$ | $(-1 - 30)x = -31x$ |
| $(x - 1)(10x - 3)$ | $(-3 - 10)x = -13x$ |
| $(2x - 3)(5x - 1)$ | $(-2 - 15)x = -17x$ |
| $(2x - 1)(5x - 3)$ | $(-6 - 5)x = -11x \leftarrow$ |

**Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.**

**1.** $2x^2 + 5x + 2$ **2.** $2n^2 - 7n + 3$ **3.** $5y^2 - 9y - 2$ **4.** $3a^2 + 7a + 2$

**5.** $4y^2 - 5y + 1$ **6.** $2a^2 + 11a + 5$ **7.** $5a^2 - 11a + 2$ **8.** $7y^2 - 9y + 2$

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## 5–9 Factoring Pattern for $ax^2 + bx + c$ *(continued)*

**Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.**

| | | | |
|---|---|---|---|
| **9.** $2k^2 - 5k - 1$ | **10.** $12k^2 - 8k + 1$ | **11.** $4x^2 + 17x - 15$ | **12.** $2a^2 + 7a + 5$ |
| **13.** $8y^2 + 6y - 9$ | **14.** $9x^2 + 3x - 2$ | **15.** $7k^2 - 11k - 6$ | **16.** $4u^2 - 8u - 5$ |

**Example 3** Factor $5 - 7x - 6x^2$.

**Solution**

$$
\begin{aligned}
5 - 7x - 6x^2 &= -6x^2 - 7x + 5 && \text{Arrange the terms by decreasing degree.}\\
&= (-1)(6x^2 + 7x - 5) && \text{Factor } -1 \text{ from each term.}\\
&= (-1)(2x - 1)(3x + 5) && \text{Factor the resulting trinomial.}\\
&= -(2x - 1)(3x + 5)
\end{aligned}
$$

*Note:* If you factor $5 - 7x - 6x^2$ directly, you will get $(5 + 3x)(1 - 2x)$. Since $(1 - 2x) = -(2x - 1)$, the two answers are equivalent.

**Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.**

| | | |
|---|---|---|
| **17.** $10 - 9y - 2y^2$ | **18.** $10 - x - 3x^2$ | **19.** $3 - x - 10x^2$ |
| **20.** $3 - 7x - 6x^2$ | **21.** $10 - u - 2u^2$ | **22.** $5 + 8x - 4x^2$ |

**Example 4** Factor $5a^2 + 2ab - 7b^2$.

**Solution**

$$
\begin{aligned}
5a^2 + 2ab - 7b^2 &= (a \qquad)(5a \qquad) && \text{Write the factors of } 5a^2.\\
&= (a - ?)(5a + ?) && \text{Test possibilities.}\\
&= (a - b)(5a + 7b)
\end{aligned}
$$

*Note:* If you write $(a + ?)(5a - ?)$ as the second step, you will not find a combination of factors that produces the desired linear term.

**Factor. Check by multiplying the factors.**

| | | |
|---|---|---|
| **23.** $x^2 - xy - 20y^2$ | **24.** $4a^2 - 4ab - 3b^2$ | **25.** $3a^2 - 5ab - 12b^2$ |
| **26.** $5a^2 + 2ab - 7b^2$ | **27.** $2x^2 - xy - 3y^2$ | **28.** $8y^2 - 6yz - 9z^2$ |

## Mixed Review Exercises

**Factor.**

| | | |
|---|---|---|
| **1.** $x^2 - 196$ | **2.** $x^2 - 7x + 12$ | **3.** $r^2 - 5r - 36$ |
| **4.** $c^2 - 10c + 25$ | **5.** $9y^2 - 121x^2$ | **6.** $4a^2 - 25$ |
| **7.** $y^2 + 13y + 36$ | **8.** $p^2 + 14p + 49$ | **9.** $9y^2 + 12y + 4$ |
| **10.** $m^2 - m - 56$ | **11.** $n^2 + 13n + 36$ | **12.** $b^2 - 3b - 54$ |